THE POLYTROPIC PROCESS OF CHANGE OF STATE OF A GAS-SOLIDS SYSTEM

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The polytropic contraction of a mixture of an ideal gas and solid particles is examined. The obtained relationship is compared with experimental data.

It has been suggested that a two-component gas-solids system can be used as a thermodynamic working medium [1–3]. Such media are of great promise in that they provide a means of increasing the thermal efficiency of heat power plants by virtue of the fact that the processes of contraction and expansion become almost isothermal.

In El'perin and Minkov's paper [2] some thermodynamic cycles were analyzed and an expression for the polytropic exponent was obtained. This expression, however, ignored several very important factors which affect the nature of the process. These include the rate of heat transfer between the particles and gas, the time of the process, and so on. In this paper we attempt to take these factors into account and obtain a more general expression for the polytropic exponent.

We consider a system consisting of an ideal gas with solid particles suspended in it. We assume that the system is adiabatically isolated; the volume of the particles during contraction (expansion) remains constant; the particles are spherical, are uniformly distributed throughout the volume, and do not interact with one another; no chemical reactions take place, and there is no heat transfer.

From the first law of thermodynamics we can write for the gas,

$$M\frac{dq}{d\tau}=M\frac{du}{d\tau}+Mp\frac{dv}{d\tau}, \tag{1}$$

and for the particles

$$M_s\frac{dq_s}{d\tau}=M_s\frac{di_s}{d\tau}. \tag{2}$$

Since the system is assumed to be adiabatically isolated, the left side of Eq. (1) is equal to the left side of Eq. (2) with the opposite sign

$$M\frac{dq}{d\tau}=-M_s\frac{dq_s}{d\tau}. \tag{3}$$

Introducing the symbol $\mu = M_s/M$ and using relationship (3) we obtain

$$-\mu\frac{di_s}{d\tau}=\frac{du}{d\tau}+p\frac{dv}{d\tau}=\frac{di}{d\tau}-v\frac{dp}{d\tau}. \tag{4}$$

We write the last expression in the form of two equations:

$$-\mu\frac{di_s}{d\tau}-\frac{du}{d\tau}=p\frac{dv}{d\tau}, \tag{4a}$$

$$-\mu\frac{di_s}{d\tau}-\frac{di}{d\tau}=-v\frac{dp}{d\tau}. \tag{4b}$$

Regarding the enthalpy of the particles as dependent on the time through the temperature of the gas, i.e.,

$$i_s=f[t(\tau)],$$

we obtain

$$\frac{di_s}{d\tau}=\frac{di_s}{dt}\frac{dt}{d\tau}=c_s\frac{dt_s}{dt}\frac{dt}{d\tau}. \tag{5}$$

On the other hand, for an ideal gas,

$$\frac{du}{d\tau}=c_v\frac{dt}{d\tau},\quad \frac{di}{d\tau}=c_p\frac{dt}{d\tau}. \tag{6}$$

Substituting relationships (5) and (6) in (4a) and (4b), we write:

$$-\left(\mu c_s\frac{dt_s}{dt}+c_v\right)\frac{dt}{d\tau}=p\frac{dv}{d\tau}, \tag{6a}$$

$$-\left(\mu c_s\frac{dt_s}{dt}+c_p\right)\frac{dt}{d\tau}=-v\frac{dp}{d\tau}. \tag{6b}$$

Dividing the second equation by the first, we obtain

$$\left(\mu c_s\frac{dt_s}{dt}+c_p\right)\Big/\left(\mu c_s\frac{dt_s}{dt}+c_v\right)=$$

$$=-\frac{v}{p}\frac{dp}{d\tau}d\tau\Big/\frac{dv}{d\tau}d\tau. \tag{7}$$

The contraction (expansion) of a two-component medium is polytropic with a variable polytropic index, the instantaneous value of which is

$$-\frac{vdp}{pdv}=n.$$

In view of this and also the fact that $c_p/c_v = k$, we obtain from Eq. (7) the following expression for the polytropic exponent:

$$n=\left(k+\mu\frac{c_s}{c_v}\frac{dt_s}{dt}\right)\Big/\left(1+\mu\frac{c_s}{c_v}\frac{dt_s}{dt}\right). \tag{8}$$

In the case of absence of solid particles, i.e., $\mu = 0$, $n = k$, and when $\mu \to \infty$, $n \to 1$.

When the temperatures of the gas and particles are equal ($t_s = t$, $dt_s = dt$) we obtain from expression (8) the formula proposed by El'perin and Minkov [2], i.e.,

$$n_0=\left(k+\mu\frac{c_s}{c_v}\right)\Big/\left(1+\mu\frac{c_s}{c_v}\right). \tag{9}$$

In the real thermodynamic process the presence of a thermal resistance external to the particles has the result that the rate of change of temperature of the particles is much lower than that of the temperature of the compressible medium (gas). The difference depends on the rate of heat transfer between the particles and the gas. Thus, other conditions being equal, at the initial instant of the process $dt_s/dt = 0$ and $n = k$, while at the end of the process, if $t_s = t$, then $n = n_0$.

In other words, in the real process the mean value of the polytropic exponent must satisfy the condition

$$n_0 \leqslant \bar{n} \leqslant k. \tag{10}$$

We write

$$\frac{dt_s}{dt} = \varphi(\tau); \tag{11}$$

$$\varphi(0) = 0, \ \varphi(\tau_\infty) = 1. \tag{11a}$$

To determine $\varphi(\tau)$ we consider the heat transfer between the spherical particles and the gas when $\mathrm{Bi} \to 0$. The heat transmitted from the gas to the particles changes the heat content of the latter, i.e.,

$$\mu c_s \frac{dt_s}{d\tau} = \alpha (t - t_s) 4\pi R_s^2 N_0, \tag{12}$$

where N_0 is the number of particles in 1 kg of gas, and

$$\mu = \rho_s \frac{4}{3} \pi R_s^3 N_0. \tag{13}$$

Substituting the value of N_0 from (13) in (12), we obtain

$$\frac{dt_s}{d\tau} = \frac{3\alpha}{c_s \rho_s R_s} (t - t_s). \tag{14}$$

We differentiate the left and right sides Eq. (14) with respect to the gas temperature. Then

$$\frac{d}{dt}\left(\frac{dt_s}{d\tau}\right) = \frac{3\alpha}{c_s \rho_s R_s}\left(1 - \frac{dt_s}{dt}\right). \tag{15}$$

If we expand $t(\tau)$ in a power series

$$t(\tau) = \sum_0^n a_n \tau^n$$

and restrict ourselves to the linear approximation, the operations of differentiation with respect to t and τ in the left side of Eq. (15) commute, i.e.,

$$\frac{d}{dt}\left(\frac{dt_s}{d\tau}\right) = \frac{d}{d\tau}\left(\frac{dt_s}{dt}\right). \tag{16}$$

Using relationship (16) and Eq. (11) we obtain the following differential equation to determine the function $\varphi(\tau)$:

$$\frac{d\varphi}{d\tau} = \frac{3\alpha}{c_s \rho_s R_s}(1 - \varphi). \tag{17}$$

Separating the variables and integrating with due regard to boundary conditions (11a), we obtain

$$\varphi(\tau) = 1 - \exp\left(-\frac{3\alpha}{c_s \rho_s R_s}\tau\right). \tag{18}$$

In this case the expression for the polytropic exponent takes the form

$$n = \left\{k + \mu \frac{c_s}{c_v}\left[1 - \exp\left(-\frac{3\alpha}{c_s \rho_s R_s}\tau\right)\right]\right\} \times \left\{1 + \mu \frac{c_s}{c_v}\left[1 - \exp\left(-\frac{3\alpha}{c_s \rho_s R_s}\tau\right)\right]\right\}^{-1}. \tag{19}$$

The mean value of the polytropic exponent during the process is

$$\bar{n} = \frac{1}{\tau}\int_0^\tau n(\tau)\, d\tau. \tag{20}$$

Integrating and carrying out some algebraic transformations we obtain the final expression for the mean value of the polytropic exponent:

$$\bar{n} = \left(k + \mu \frac{c_s}{c_v}\right) \Big/ \left(1 + \mu \frac{c_s}{c_v}\right) + \frac{1}{3\mathrm{Bi\,Fo}} \times \ln\left\{1 + \mu \frac{c_s}{c_v}\left[1 - \exp(-3\mathrm{Bi\,Fo})\right]\right\} \times \left[\left(k + \mu \frac{c_s}{c_v}\right) \Big/ \left(1 + \mu \frac{c_s}{c_v}\right) - 1\right]. \tag{21}$$

In the limiting case, when $\tau \to \infty$, relationship (21) becomes expression (9).

The values of the polytropic exponent calculated from formula (21) were compared with the experimentally obtained values. The apparatus on which the experiment was conducted was described in [4]. A mixture of air and particles of natural graphite (particle size 10μ) was compressed in a rotary compressor. The concentration, temperature, and pressure of the mixture were measured in the experiments. The time of the contraction process was determined from the travel of the piston for a known number of revolutions of the compressor. The coefficient of heat transfer between the gas and graphite particles was evaluated from the formula

$$\alpha \approx 1.8\lambda / d_s, \tag{22}$$

which is valid for round particles in the case of undetached flow [3]. A comparison of the values of the polytropic exponent calculated from formula (21) with the experimentally obtained values showed that the differences were not more than 3%. For instance, at graphite concentrations of 4.5, 10.9 and 13.3 kg/kg the polytropic exponent was, respectively, 1.072, 1.067, and 1.046 by experiment, and 1.075, 1.050, and 1.037 by calculation.

Thus, the obtained expression for the polytropic exponent can be used to calculate and analyze the cycles of heat power plants in which the working medium is composed of a gas and solid particles.

NOTATION

M is the mass; q is the heat flux; τ is the time of process; u is the internal energy of gas; p is the pressure; v is the specific volume; i is the enthalpy; μ is the mass concentration of solid phase; t is the temperature; c_s, c_p, c_v are the specific heat of particle material, specific heat of gas for p = const and v = = const, respectively; n is the polytropic exponent; k is the adiabatic exponent; α is the coefficient of heat transfer between gas and particles; R_s is the particle radius; ρ is the density; λ is the thermal conductivity; Bi is the Biot number; Fo is the Fourier number. The subscript s indicates that the value relates to the solid phase.

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